DOOM Puts and Credit Protection

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Bear Put Spreads

- A bear put spread arises by buying one out-of-the-money (OTM) put and selling one deeper OTM put on the same stock with the same maturity.
- For example with the stock price at $10, buy the GE put struck at $5 and sell the GE put struck at $2.50, with both maturing Jan. 2011.
- The minimum payoff from a bear put spread is zero and the maximum payoff is the difference in strikes.
- Under what stock price dynamics does the current price of a bear put spread equal zero?
- Under what stock price dynamics does the current price of a bear put spread approach the difference in strikes?

Reading Bear Put Spreads

- If the stock price can never get below the higher strike of a bear put spread, then the bear put spread is worthless.
- Suppose that we equate default with the stock price jumping from above the higher strike to below the lower strike and then staying below the lower strike until expiry. In such a scenario, both puts would be exercised right after the down jump.
- As the probability of the immediate occurrence of this event approaches one, the pre-default price of a bear put spread approaches the difference in strikes.
- If we assume that the stock price is always above the higher strike prior to any default, then the higher is the pre-default price of a bear put spread, the more valuable is a claim paying one dollar upon default.
- In fact, under the above assumptions, the ratio of a bear put spread to the difference in strikes is the value of a claim paying one dollar upon default.

Example of a Scaled Bear Put Spread

- With the stock price at \$10, suppose that we buy $2/5$ of the Jan 2011 GE put struck at \$5 and sell $2/5$ of the Jan. 2011 put struck at \$2.50.
- If the stock price stays above \$5 until Jan. 2011, then both puts expire worthless. If a default prior to Jan. 2011 causes the stock price to jump from above \$5 to below \$2.50 permanently, then both puts should be exercised at the default time. We get $2/5$ of \$2.50 = \$1 at such a default time.
- It follows that the cost today of buying $2/5$ of a put struck at \$5 and selling $2/5$ of a put struck at \$2.50 is the cost today of creating a claim paying \$1 at the default time and zero otherwise.
- We refer to this claim as a Unit Recovery Claim (URC).

Formal Theory: Default Corridor

- With the stock price at $S > 0$, suppose there exists a barrier $B \in (0, S)$ that the stock price stays above before default.
- Suppose further that there exists another barrier $A \in [0, B)$ which the stock price drops below and remains under, at and after default.
- Mnemonic: B is below S before default; A is above S at and after default.
- *Why Might a Default Corridor Exist?*
  - Strategic default: Debt holders have incentives to induce default before the equity value vanishes ($B > 0$). (Anderson, Sundaresan (96), Mella-Barral, Perraudin (97), Fan, Sundaresan (00), Broadie, Chernov, Sundaresan (07), Carey, Gordy (07), Hackbarth, Hennessy, Leland (07).)
  - Sudden default generates sudden drops in equity value ($A < B$)due to legal fees, liquidation costs, and loss of continuation value on projects...

Scaled Bear Put Spreads and Credit Protection

- We assume that a default corridor $[A, B]$ exists inside $[0, S)$.
- Let $P_t(K, T)$ be the time t price of an American put struck at K and maturing at T. Then for any two strikes $K_1 \in [A, B)$ and $K_2 \in (K_1, B]$, the scaled bear put spread, $U^p(t, T) \equiv (P_t(K_2, T) - P_t(K_1, T))/(K_2 - K_1)$ is the time t cost of replicating a Unit Recovery Claim (URC), which is defined to pay:
  - \$1 at the default time τ if the company defaults prior to the option expiry T and \$0 otherwise.

Butterfly Spreads

- Recall that a butterfly spread is a static position in co-terminal options defined by three strikes. For any pair of outer strikes $K_0 \geq 0$ and $K_2 > K_0$, a middle strike $K_1 \in (K_0, K_2)$ defines a probability $\lambda \equiv \frac{K_2 - K_1}{K_2 - K_0} \in [0, 1]$.
- A butterfly spread arises by selling one middle strike put and buying λ lower strike puts and $1 - \lambda$ higher strike puts. The net cost is $\lambda P_t(K_0, T) - P_t(K_1, T) + (1 - \lambda) P_t(K_2, T)$.
- A butterfly spread of co-terminal American puts always has a nonnegative price since the outer puts can always be exercised when the middle strike put is exercised.
- If the price of a butterfly spread is positive, it is because the market thinks there is some value to not exercising either of the outer strike puts exactly when the middle strike put is exercised.
- Since the zero strike put is worth zero, one can always form a butterfly spread using the zero strike put and the two puts struck at K_1 and K_2. Under what stock price dynamics would this butterfly be free?

The DOOM Put

- Setting $K_0 = 0$ and $P_t(K_0, T) = 0$, the cost of forming a butterfly spread with two positive strikes $K_1 > 0$ and $K_2 > K_1$ is $-P_t(K_1, T) + (1 - \lambda)P_t(K_2, T)$, where $\lambda \equiv \frac{K_2 - K_1}{K_2} \in (0, 1)$.
- If this butterfly is free, then no arbitrage implies that the stock price must always be above K_2 prior to the default time τ and then jump to 0 at τ.
- If a free butterfly is observed, then a URC is created more simply by buying either $\frac{1}{K_1}$ of the K_1 strike put or $\frac{1}{K_2}$ of the K_2 strike put.
- If the above butterfly costs money, then the stock price S can get below K_2 prior to τ. However, it may be that $S > K_1 > 0$ prior to τ and that S jumps to zero at τ.
- Under this assumption, a URC is created by buying $\frac{1}{K_1}$ of the K_1 strike put.
- Recall our default corridor $[A, B]$ assumption had $0 \leq A \leq K_1 < K_2 \leq B < S$. A positive $(0, K_1, K_2)$ butterfly implies $K_2 > B$.
- The latter S dynamics become consistent with our assumption if $A = 0$ & if the 2 strikes in the scaled bear put spread are $(0, K_1)$, instead of (K_1, K_2). Then, $U^p(t, T)$ is the DOOM put price $P_t(K_1, T)$ divided by K_1.

Extracting URC's from CDS's?

- By assuming that there exists a strike $K_1 > 0$ which the stock price S exceeds prior to the default time τ, and that S jumps to 0 at τ, we have shown that $U^p(t, T) = P_t(K_1, T)/K_1$.
- Can we also make enough assumptions so that the value of a Unit Recovery Claim can be extracted from a CDS spread of the same maturity as the put?
- The standard market model that Bloomberg has implemented on the CDSW function assumes that CDS are priced as if the recovery rate on the underlying bond is constant and as if the future instantaneous interest rate and the future risk-neutral default arrival rate are piecewise constant functions of time.
- CDSW uses this model to strip the (risk-neutral) default probabilities from the term structure of CDS. The model can also be used to strip the term structure of URC's from the same data. In the current environment of low interest rates, the URC is just slightly below the implied default probability (due to slight discounting for time value of money).
- Let's see how the two URC's compare for GE.

Extracting a URC from a CDS?

- Suppose we continue to assume a constant bond recovery rate R^b.
- Suppose we further assume that the future instantaneous interest rate and the future risk-neutral default arrival rate are constant at r and λ respectively, rather than piecewise constant functions of time.
- Then the model implies that the value of a URC of maturity T at time t can be extracted from a contemporaneous CDS quote $k_t^c(T)$ of the same maturity:

$$U^c(t,T) = \lambda^c \frac{1-e^{-(r+\lambda^c)(T-t)}}{r+\lambda^c},$$

where $\lambda^c \equiv k_t^c(T)/(1-R^b)$.

- In reality, rates and CDS spreads will vary over both time and term, so we just conveniently calculate:

$$U^c(t,T) = \lambda^c(t,T) \frac{1-e^{-(r(t,T)+\lambda^c(t,T))(T-t)}}{r(t,T)+\lambda^c(t,T)},$$

where $\lambda^c(t,T) \equiv k_t^c(T)/(1-R^b)$.

Comparing Unit Recovery Claim Values

- Fortunately, the assumptions used to imply the URC from the DOOM put price are not inconsistent with the assumptions used to imply the URC from the single CDS spread of the same maturity.
- Making both sets of assumptions, we can compare the implied URC values for the same name and at the same time and term.
- If the assumptions hold and the implied prices differ, then a model-based cross market arbitrage arises.
- If the assumptions hold and the implied prices are the same, then:
  - the DOOM put is being priced off the CDS independently of the exact stock price level S and independently of historical volatility and the ATM implied volatility.
  - Conversely, the CDS is being priced off the DOOM put, independently of the price of the underlying bond and independently of the company's leverage.

Empirical Analysis: Sample Selection

- Collect data on listed (so American-style) US single-name puts and CDS spreads from various sources (eg. Bloomberg, Option Metrics) on a list of companies.
  - Sample period: January 2005 to June 2007
  - Company selection criteria:
    - OptionMetrics have non-zero bid quotes on one or more put options struck more than one standard deviation below the current spot price and with maturities over 180 days.
    - Reliable CDS quotes are available at 1-, 2-, 3-year maturities.
    - The average CDS spreads at 1-year maturity is over 30bps.

List of selected companies

Equity Ticker	Cusip Number	Company Name
AMR	00176510	American Airline
CTB	21683110	Cooper Tire & Ribber
DDS	25406710	Dillard's Inc.
EK	27746110	Eastman Kodak Co
F	34537086	Ford Motor Co
GM	37044210	General Motors Corp
GT	38255010	Goodyear Tire & Rubber Co
KBH	48666K10	KB Home

Infer the URC value from American Put and CDS

- For all maturities > 180 days, we used the American put with the lowest strike K with non-zero bid.
- Take CDS quotes available at fixed maturities (1, 2, 3 years). Linear interpolation to obtain CDS spread at the longest option maturity.
- From the interpolated CDS spread $k(t, T)$, infer the default arrival rate based on constant interest rate and default rate assumptions and an assumed 40% bond recovery: $\lambda^c(t, T) = k^c(t, T)/(1 - R^b)$.
  - One can use default recovery swap to fix the recovery.
- Compute the unit recovery claim value according to,

$$U^c(t, T) = \lambda^c(t, T) \frac{1 - e^{-(r(t,T)+\lambda^c(t,T))(T-t)}}{r(t, T) + \lambda^c(t, T)}.$$

Summary Statistics for Unit Recovery Claim Values

Ticker	U^p			U^c			Cross-market
	Mean	Std	Auto	Mean	Std	Auto	Correlation
AMR	0.116	0.073	0.982	0.265	0.167	0.995	0.933
CTB	0.087	0.039	0.964	0.052	0.042	0.993	0.369
DDS	0.063	0.026	0.981	0.032	0.018	0.990	0.709
EK	0.043	0.019	0.969	0.037	0.019	0.989	0.869
F	0.103	0.044	0.967	0.136	0.066	0.989	0.806
GM	0.085	0.060	0.968	0.165	0.106	0.994	0.941
GT	0.075	0.034	0.970	0.073	0.034	0.986	0.869
KBH	0.048	0.038	0.983	0.026	0.010	0.984	0.774
Average	0.077	0.042	0.973	0.098	0.058	0.990	0.784

- Similar (but not identical) mean magnitudes for the unit recovery values from the two markets.
- High cross-market correlations.

Why Do Mean URC Levels Differ?

- The mean put implied URC differs from the mean CDS implied URC. Why?
- There can be at least 3 reasons:
  1. Incorrect Model Assumptions: eg., the stock price can get below any level before default, or bond recovery rates and interest rates are random and correlated with each other and default times.
  2. Correct Model Assumptions but Incorrect Parameter Settings: eg. the bond recovery rate is constant, but is not exactly 0.4. Similarly, the post default stock price is a constant below K_1, but is not exactly zero.
  3. Correct Model Assumptions, Correct Parameter Settings and Cross-Market Arbitrage Exists.
- For some names eg. American Airlines and General Motors, the correlation between the put-implied URC and the CDS-implied URC is above 90%. Yet for those names, the put-implied URC's have a different time series mean than the CDS-implied URC. To test the second explanation, we can use the model errors themselves to correct the parameter settings, without necessarily removing any cross-market arbitrage opportunities that may exist.

Contemporaneous Regressions on the two URC series

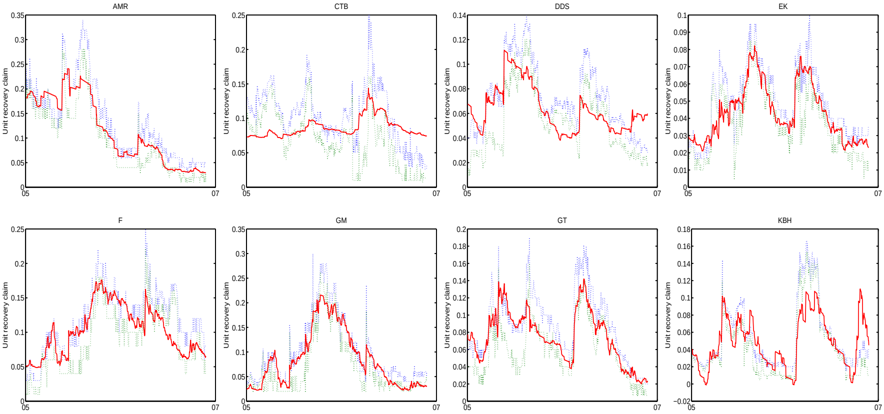
$U_t^p = a + bU_t^c + \mathcal{U}_t,$

Ticker	a		b		R^2
AMR	0.008	(1.42)	0.406	(-17.32)	0.870
CTB	0.069	(4.53)	0.343	(-4.47)	0.136
DDS	0.029	(3.55)	1.049	(0.24)	0.502
EK	0.012	(3.77)	0.847	(-2.44)	0.755
F	0.030	(2.44)	0.531	(-6.71)	0.650
GM	-0.004	(-0.64)	0.539	(-10.44)	0.885
GT	0.010	(1.88)	0.890	(-1.09)	0.756
KBH	-0.025	(-2.03)	2.858	(2.85)	0.599
Average	0.016	(1.87)	0.933	(-4.92)	0.644

- In parentheses are t-statistics against the null: $a = 0$ and $b = 1$.
- Positive slope (co-movements), high R-squared.
- $\mathcal{U}_t$ captures the unexplained component of the American put $U_t^p = P_t/K$.
  — Non-CDS driven variation in American puts.

American put bid-ask quotes and CDS

Red — CDS $(a + bU_t^c)$, Blue — American put ask, Green — American put bid.



Explain non-CDS driven daily variation in American puts

Stock price: $\Delta \mathcal{U}_t = a + b\Delta S_t + e_t$,

Ticker	a		b		R^2
AMR	0.000	(0.33)	-0.003	(-4.07)	0.026
CTB	-0.000	(-0.44)	-0.008	(-4.45)	0.080
DDS	0.000	(0.66)	-0.004	(-6.29)	0.325
EK	-0.000	(-0.25)	-0.003	(-6.01)	0.100
F	-0.000	(-0.22)	-0.008	(-2.97)	0.024
GM	-0.000	(-0.14)	-0.003	(-2.44)	0.030
GT	0.000	(0.88)	-0.003	(-2.88)	0.038
KBH	-0.000	(-0.14)	-0.000	(-0.69)	0.002

There is a negative delta component in the American puts:

- Either OTM put prices are not purely driven by default risk (as captured in CDS spreads), or:
- S_t contains credit risk information absent from the current CDS quotes.

Explain non-CDS driven daily variation in American puts

Realized volatility: $\Delta\mathcal{U}_t = a + b\Delta RV_t + e_t$,

Ticker	a		b		R^2
AMR	0.000	(0.13)	0.181	(0.57)	0.001
CTB	-0.000	(-0.71)	0.129	(0.68)	0.002
DDS	-0.000	(-0.16)	-0.249	(-4.08)	0.047
EK	0.000	(0.17)	0.105	(0.91)	0.004
F	0.000	(0.30)	-0.139	(-0.87)	0.001
GM	0.000	(0.04)	0.058	(0.37)	0.000
GT	0.000	(0.15)	-0.049	(-0.64)	0.001
KBH	-0.000	(-0.09)	-0.015	(-0.11)	0.000

Nothing much here. Given the CDS quotes, OTM put prices do not respond to changes in RV, as our theory predicts.

Explain non-CDS driven daily variation in American puts

At-the-money implied volatility: $\Delta \mathcal{U}_t = a + b\Delta ATMV_t + e_t$,

Ticker	a		b		R^2
AMR	0.000	(0.45)	0.213	(3.23)	0.037
CTB	-0.000	(-0.77)	0.362	(2.81)	0.137
DDS	0.000	(0.84)	0.384	(4.61)	0.188
EK	0.000	(0.33)	0.227	(3.70)	0.083
F	-0.000	(-0.05)	0.222	(3.53)	0.067
GM	-0.000	(-0.13)	0.241	(2.78)	0.066
GT	0.000	(0.78)	0.409	(5.25)	0.203
KBH	-0.000	(-0.05)	0.262	(2.09)	0.027

Slopes are all positive and significant. There is a risk premium priced into both options that is not priced into the contemporaneous CDS quotes. This risk premium could compensate for either:

- stochastic instantaneous volatility when minimum pre-default stock price vanishes, or
- default risk - i.e. a default risk premium is priced into both options but not CDS.

Predicting implications: Hypotheses

- H1: $\mathcal{U}_t \equiv U_t^p - (a + bU_t^c)$ is purely due to transient noise in the options market.
  $\Rightarrow$ A positive $\mathcal{U}_t$ predicts a future decline in the American put value.
- H2: $\mathcal{U}_t$ reflects credit risk information from the options market that has not yet shown up in the current CDS quotes.
  $\Rightarrow$ A positive $\mathcal{U}_t$ predicts a future increase in the CDS spread.

Error-correction regressions:

$$\Delta U_{t+\Delta t}^p = \alpha^p + \beta^p \mathcal{U}_t + e_{t+\Delta t}, \quad \Delta U_{t+\Delta t}^c = \alpha^c + \beta^c \mathcal{U}_t + e_{t+\Delta t}$$

- Under H1, $\beta^p < 0$
- Under H2, $\beta^c > 0$.

Predicting Put Prices and CDS Spreads over 1-day horizon

$\Delta U^p_{t+\Delta t} = \alpha^p + \beta^p \mathcal{U}_t + e_{t+\Delta t}, \quad \Delta U^c_{t+\Delta t} = \alpha^c + \beta^c \mathcal{U}_t + e_{t+\Delta t},$
$\Delta t = 1$ day.

Ticker	β^p		R^2	β^c		R^2
AMR	-0.070	(-2.77)	0.033	0.037	(1.94)	0.012
CTB	-0.015	(-1.44)	0.005	-0.004	(-0.42)	0.001
DDS	-0.021	(-2.64)	0.012	-0.001	(-0.38)	0.000
EK	-0.093	(-4.30)	0.052	0.014	(2.13)	0.005
F	-0.079	(-2.90)	0.050	-0.028	(-2.72)	0.015
GM	-0.127	(-3.47)	0.054	0.051	(1.70)	0.019
GT	-0.065	(-3.65)	0.033	0.003	(0.22)	0.000
KBH	-0.007	(-1.22)	0.002	0.007	(2.06)	0.013
Average	-0.060	(-2.80)	0.030	0.010	(0.57)	0.008

- H1: β^p is significantly negative for 6 of the 8 companies. R^2 averages at 3%.
- H2: β^c is significantly positive for 4 of 8 companies. R^2 averages at 0.8%.

Predicting Put Prices and CDS Spreads over 7-day horizon

$\Delta U^p_{t+\Delta t} = \alpha^p + \beta^p \mathcal{U}_t + e_{t+\Delta t}, \quad \Delta U^c_{t+\Delta t} = \alpha^c + \beta^c \mathcal{U}_t + e_{t+\Delta t},$
$\Delta t = 7$ days.

Ticker	β^p		R^2	β^c		R^2
AMR	-0.174	(-2.48)	0.078	0.166	(1.70)	0.036
CTB	-0.056	(-1.19)	0.015	-0.004	(-0.08)	0.000
DDS	-0.087	(-2.54)	0.051	-0.013	(-0.52)	0.003
EK	-0.296	(-5.90)	0.170	0.021	(0.59)	0.001
F	-0.204	(-2.64)	0.113	-0.144	(-2.52)	0.039
GM	-0.171	(-2.74)	0.043	0.213	(1.62)	0.035
GT	-0.188	(-2.48)	0.079	-0.015	(-0.19)	0.001
KBH	-0.002	(-0.07)	0.000	0.053	(2.76)	0.075
Average	-0.147	(-2.51)	0.069	0.035	(0.42)	0.024

- H1: β^p is significantly negative for 6 of the 8 companies. R^2 averages at 6.9%.
- H2: β^c is significantly positive for 3 of 8 companies. R^2 averages at 2.4%.

Predicting Put Prices and CDS Spreads over 30-day horizon

$\Delta U^p_{t+\Delta t} = \alpha^p + \beta^p \mathcal{U}_t + e_{t+\Delta t}, \quad \Delta U^c_{t+\Delta t} = \alpha^c + \beta^c \mathcal{U}_t + e_{t+\Delta t},$
$\Delta t = 30$ days.

Ticker	β^p		R^2	β^c		R^2
AMR	-0.594	(-3.69)	0.240	-0.333	(-1.30)	0.031
CTB	-0.143	(-1.06)	0.028	-0.010	(-0.09)	0.000
DDS	-0.288	(-3.13)	0.161	-0.067	(-0.84)	0.017
EK	-0.963	(-9.42)	0.415	-0.177	(-1.26)	0.020
F	-0.583	(-3.34)	0.234	-0.487	(-2.62)	0.095
GM	-0.241	(-1.08)	0.023	0.724	(1.73)	0.064
GT	-0.598	(-2.29)	0.186	-0.133	(-0.65)	0.011
KBH	0.046	(0.55)	0.008	0.175	(3.77)	0.171
Average	-0.421	(-2.93)	0.162	-0.038	(-0.16)	0.051

- H1: β^p is significantly negative for 5 of the 8 companies. R^2 averages at 16.2%.
- H2: β^c is significantly positive for 2 of 8 companies. R^2 averages at 5.1%.

Prediction Implications: Summary

Error-correction regressions:

$$\Delta U^p_{t+\Delta t} = \alpha^p + \beta^p \mathcal{U}_t + e_{t+\Delta t}, \quad \Delta U^c_{t+\Delta t} = \alpha^c + \beta^c \mathcal{U}_t + e_{t+\Delta t}$$

- R-squares from the first regression is higher than that from the second regression.
- There are more significantly negative β^p than significantly positive β^c.
- Implications:
  - The credit risk information mainly flows from the CDS market to the American put options market.
  - For a few companies, the credit risk information also flows the other way around.

Concluding Remarks

- We propose a theory that links the price of a deep out-of-the-money (DOOM) put to a CDS of the same term.
- The portion of the theory that extracts the URC value from the DOOM put price is both simple and robust:
  - Simple: A scaled American put replicates the URC's payoff.
  - Robust: The replication is valid as long as the pre-default stock price is always above the DOOM put's strike, irrespective of pre-default stock price dynamics, interest rate movements, or credit risk fluctuations.
- The theoretical linkage between DOOM puts and co-terminal CDS has strong empirical support:
  - The values of the URC inferred from DOOM put prices and from CDS spreads have strong, positive correlations.
  - Deviations of the observed put implied URC's from a regression-corrected CDS-implied URC do predict future movements of prices of American put options.