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Bringing Exotic Models Home for VA Hedging – Part I

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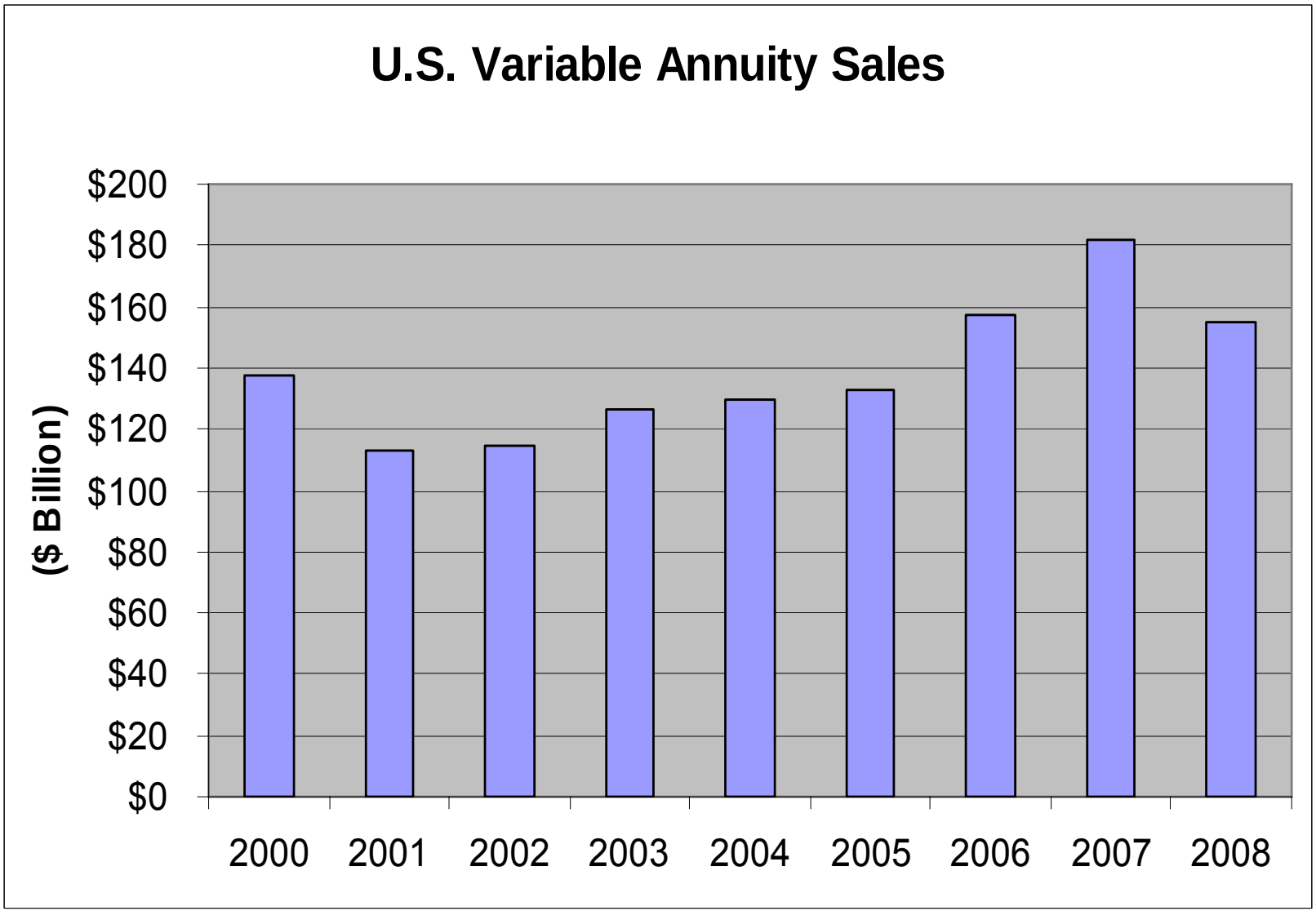
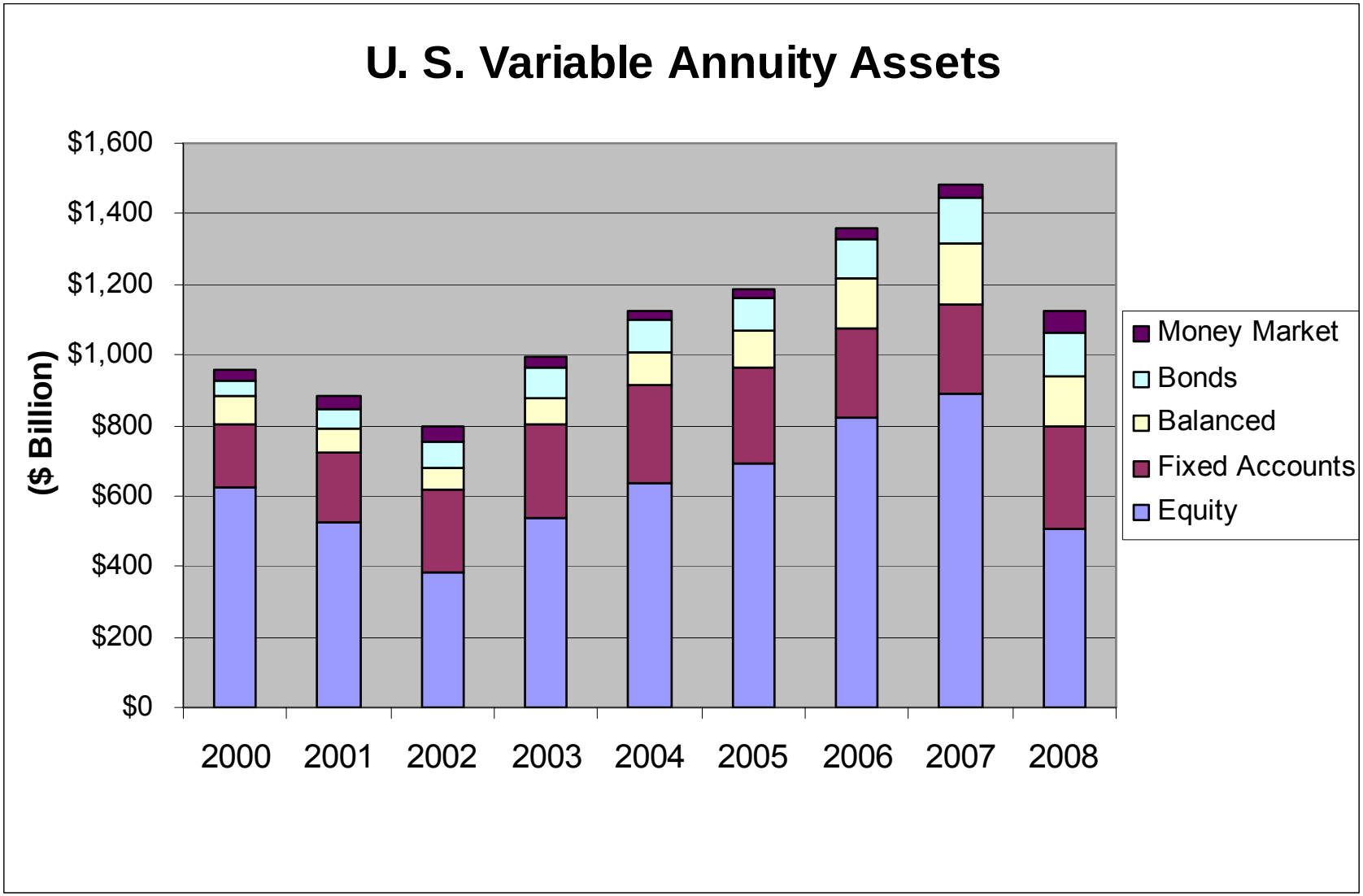


Source: Morningstar, Inc.



Source: Morningstar, Inc.

GMWB Basic Product Features

- When a GMWB is purchased, proceeds are invested at policyholders' discretion from a restricted menu of fund choices
- Depending on age at first withdrawal, policyholders can withdraw up to a maximum percentage (typically 4-6%) of their investment per year for life
- If account value is exhausted before end of contract, insurer sets up a payout annuity at the time AV goes to $0
- Fee charge ranging from 75 – 125 bps per year by most insurers

GMWB Basic Product Features

- Other features:
  - Three-year bonus – during the first three years of their contract, policyholders receive a bonus that's automatically applied to the guarantee each year until they start taking withdrawals
  - Doubler bonus – delay taking the first withdrawal for 10 years, the GMWB base will double
  - Step-up – a step-up can increase rider withdrawal benefit payments if the contract accumulated value increases. The contract accumulated value increases whenever additional premium payments are made, the division values rise with market growth, or credits are applied

The GMWB Product Structure

- An annuity (periodic guarantee withdrawals)

+

- The upside residual in the account at maturity (call option)

The Insurance Guarantee

- The account value

+

- Insurance guarantee (put option like)

Risk Considerations

- Market risks:
  - Equity
  - Interest rate
  - Volatility
  - Correlation
- Product features:
  - Fund choices – fund classes, asset allocation split / restriction (e.g. 60/40)
  - Withdrawal rate
  - Other features – bonus, step-up, doubler, etc.
  - Fee drag
- Policyholder behavior:
  - Surrender
  - Longevity / Mortality
  - Timing and level of withdrawal
  - Fund elections
- Sales mix:
  - Age
  - Gender
  - Qualified vs. non-qualified

Affecting Guarantee / Option Costs

- Equity market risk
- Interest rate risk
- Correlation of rates-equity
- Basket of underlyings (equity indices, fixed income, etc.)
- Maximum annual withdrawal rate, w%
- Bonus, step-up, doubler, etc.

Key Components of Guarantee / Option Costs

- Basket volatility
  - As it has been shown, insurance cost increases exponentially with the basket volatility,
    - which is affected by the underlying volatility of each of the funds (fund choices),
    - as well as by fund / asset allocation split and
    - Correlations
- Maximum withdrawal rate
  - Higher withdrawal rate increases the annuity component and the insurance cost to guarantee

More on Volatility and Correlation

- Volatility of the product:

$$Var_p = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i \, w_j \, \sigma_{ij} \quad ; \quad \sigma_{ij} = \rho_{i\,j} \, \sigma_i \, \sigma_j$$

$$Var_p = \sum_{i=1}^{n} \sigma_i^2 \, w_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{i\,j} \, \sigma_i \, \sigma_j \, w_i \, w_j \,, \quad \text{where } i \neq j$$

$$\sigma_p = (Var_p)^{0.5}$$

Effects on Guarantee Costs

- Assuming the following:
  - Using a fund menu of 4 fund choices
  - Various correlation assumptions
  - Assuming a return volatility of 25% for each of the fund
  - Equal fund allocations

Effects on Guarantee Costs (cont.)

By varying the correlations, the basket volatility and therefore, the guarantee costs of the rider is impacted

Correlation	Basket Volatility	Cost (%)	bp
-0.25	6.3%	0.16	1.5
0.00	12.5%	1.31	12.2
0.25	16.5%	3.39	31.6
0.50	19.8%	5.57	51.9
0.75	22.5%	7.54	70.2
1.00	25.0%	9.46	88.1

Typical Hedging Programs By the Top 20

- Dynamic Hedging Program – delta only – 9 out of 20
- Dynamic Hedging Program – beyond delta – 9 out of 20
- Static Hedging Program – 1 out of 20

Source: Towers Perrin Research

Typical Hedging Programs

- Delta Hedging
  - Futures and total return swaps
  - Works well in calm market
  - Significant risk in volatile market
- Dynamic Hedging Beyond Delta
  - Hedge delta, vega and rho
  - Equity index futures and options with various terms, and variance swaps are used to hedge delta and vega
  - Rate futures, interest rate swaps and swaptions to hedge rho
  - Major greeks of variable annuity are hedged
  - Mirror well with gain/loss of liabilities due to capital market fluctuations

Typical Hedging Programs (cont.)

- Dynamic Hedging Beyond First Order Sensitivities
  - Hedge delta, vega, rho + gamma, and convexity
  - In addition to the above instruments, exotics, such as lookbacks, hybrids, are used to hedge cross greeks and second order sensitivities
  - More precise hedge in extreme tail events
  - Higher degree of model risk
  - Liquidity and transaction costs need to be factored in
- Static Hedging
  - Customized hedge position to match embedded guarantee / option in liabilities
  - Extensive use of customized instruments and exotics
  - Highest hedge effectiveness
  - Inflexible to adjust for changes in asset portfolio and fund switching
  - Inflexible to adjust mis-estimation of actuarial settings
  - Liquidity and transaction costs need to be factored in

Exotic Option Structures

- To recap – the following are the key risk factors; they could be replicated via use of options and exotics:
  - Volatility – puts, variance swaps
  - Correlation – hybrids
  - Withdrawal – jumps
- Depending on the features, other exotics might be useful:
  - Ratchets
  - Asians
  - Lookbacks