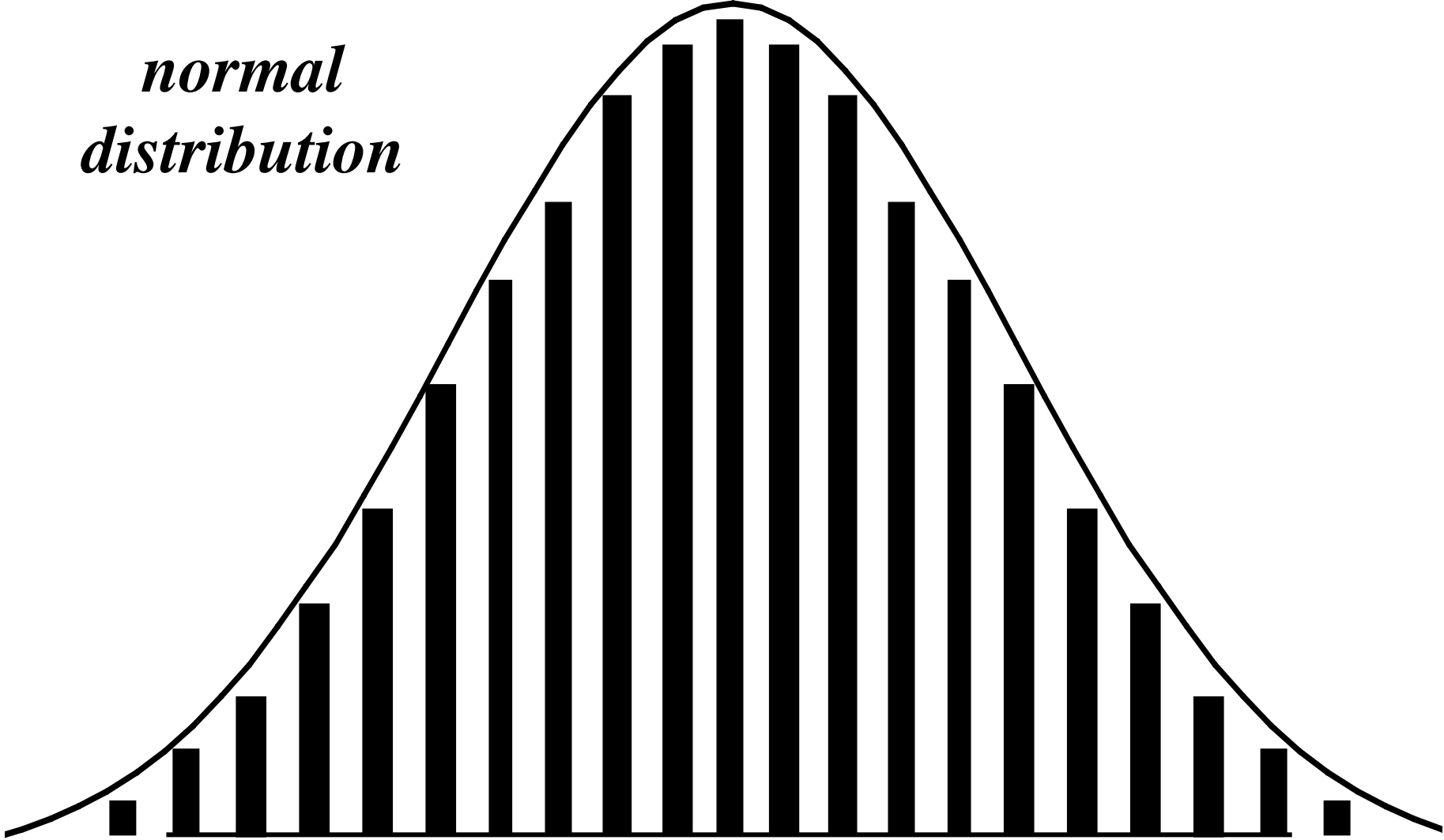




Volatility

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*normal
distribution*



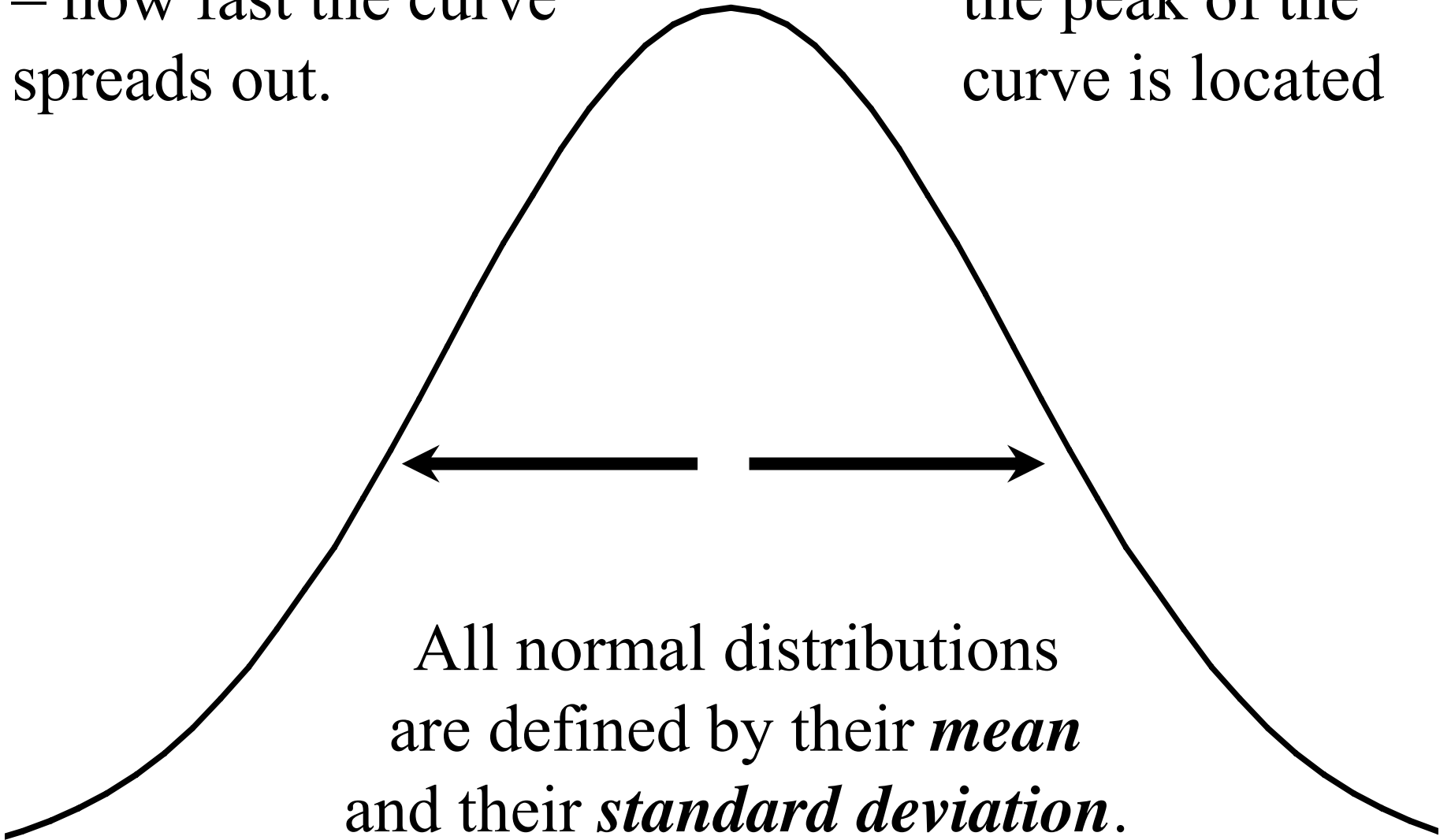
underlying prices

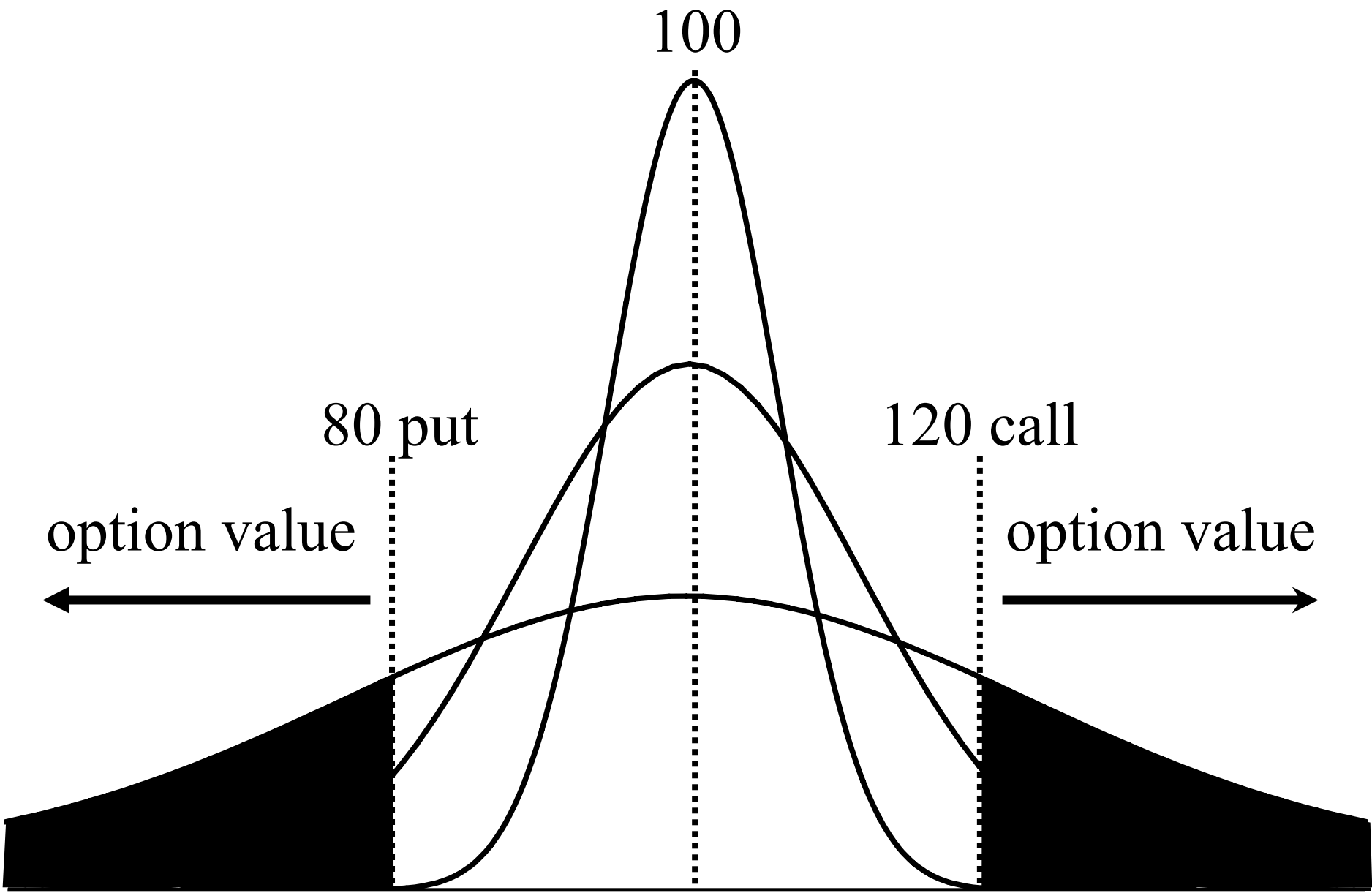
standard deviation (σ)

– how fast the curve spreads out.

mean (μ) – where

the peak of the curve is located





+1 S.D. \approx 34%

-1 S.D. \approx 34%

+2 S.D. \approx 47.5%

-2 S.D. \approx 47.5%

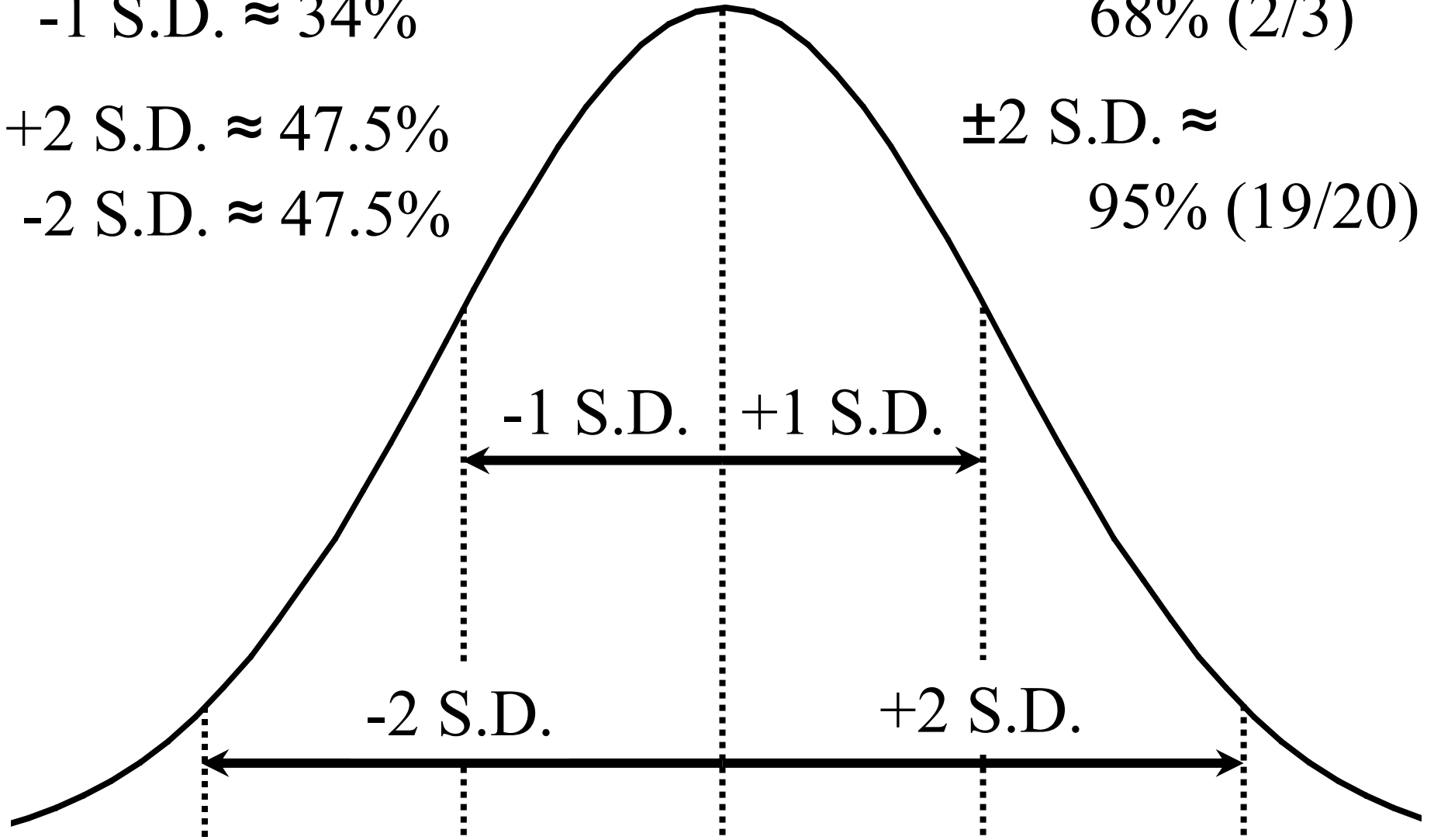
± 1 S.D. \approx

68% (2/3)

± 2 S.D. \approx

95% (19/20)

mean



We would expect to see an occurrence

- within 1 standard deviation
approx. 2 times out of 3
- within 2 standard deviations
approx. 19 times out of 20
- greater than 1 standard deviation
approx. 1 time in 3
- greater than 2 standard deviations
approx. 1 time in 20

model inputs

exercise price

time to expiration

underlying price

interest rate

volatility

(dividends)

mean?

standard
deviation?

Mean – forward price

Standard deviation – volatility

Volatility: one standard deviation,
in percent, over a one year period.

1-year forward price = 100.00

volatility = 20%

One year from now:

- 2/3 chance the contract will be between 80 and 120 ($100 \pm 20\%$)
- 19/20 chance the contract will be between 60 to 140 ($100 \pm 2*20\%$)
- 1/20 chance the contract will be less than 60 or more than 140

What does an annual volatility tell us about movement over some other time period?

$$\text{Volatility}_t = \text{Volatility}_{\text{annual}} * \sqrt{t}$$

daily standard deviation:

$$\text{volatility}_{\text{annual}} / \sqrt{256} \approx \text{volatility}_{\text{annual}} / 16$$

weekly standard deviation:

$$\text{volatility}_{\text{annual}} / \sqrt{52} \approx \text{volatility}_{\text{annual}} / 7.2$$

monthly standard deviation:

$$\text{volatility}_{\text{annual}} / \sqrt{12} \approx \text{volatility}_{\text{annual}} / 3.5$$

current price = 100.00

volatility_{daily} $\approx 20\% / 16 = 1\frac{1}{4}\%$

One trading day from now:

- 2/3 chance the contract will be between 98.75 and 101.25

$$(100 \pm 1\frac{1}{4}\%)$$

- 19/20 chance the contract will be between 97.50 and 102.50

$$(100 \pm 2 * 1\frac{1}{4}\%)$$

stock = 68.50; volatility = 42.0%

daily standard deviation

$$\approx 68.50 * 42\% / 16$$

$$= 68.50 * 2.625\% \approx \mathbf{1.80}$$

weekly standard deviation

$$\approx 68.50 * 42\% / 7.2$$

$$= 68.50 * 5.83\% \approx \mathbf{4.00}$$

stock = 68.50; volatility = 42.0%

daily standard deviation = 1.80

+0.70 +1.25 -0.95 -1.60 +0.35

Is 42% a reasonable volatility estimate?

How often do you expect to see an occurrence greater than one standard deviation?

Volatility Exercise I

For each contract and volatility below, what would be an approximate daily and weekly standard deviation:

Stock at 45.75

25%

35%

45%

50%

daily

weekly

Stock index 1242.00

30%

40%

50%

60%

daily

weekly

Volatility Exercise II

For each contract, volatility, and time interval below, what would be an approximate one standard deviation price change:

Stock price = 77.15

volatility = 47%, time = 86 days

volatility = 29%, time = 22 days

Index price = 844.00

volatility = 39%, time = 5 weeks

volatility = 15%, time = 11 weeks

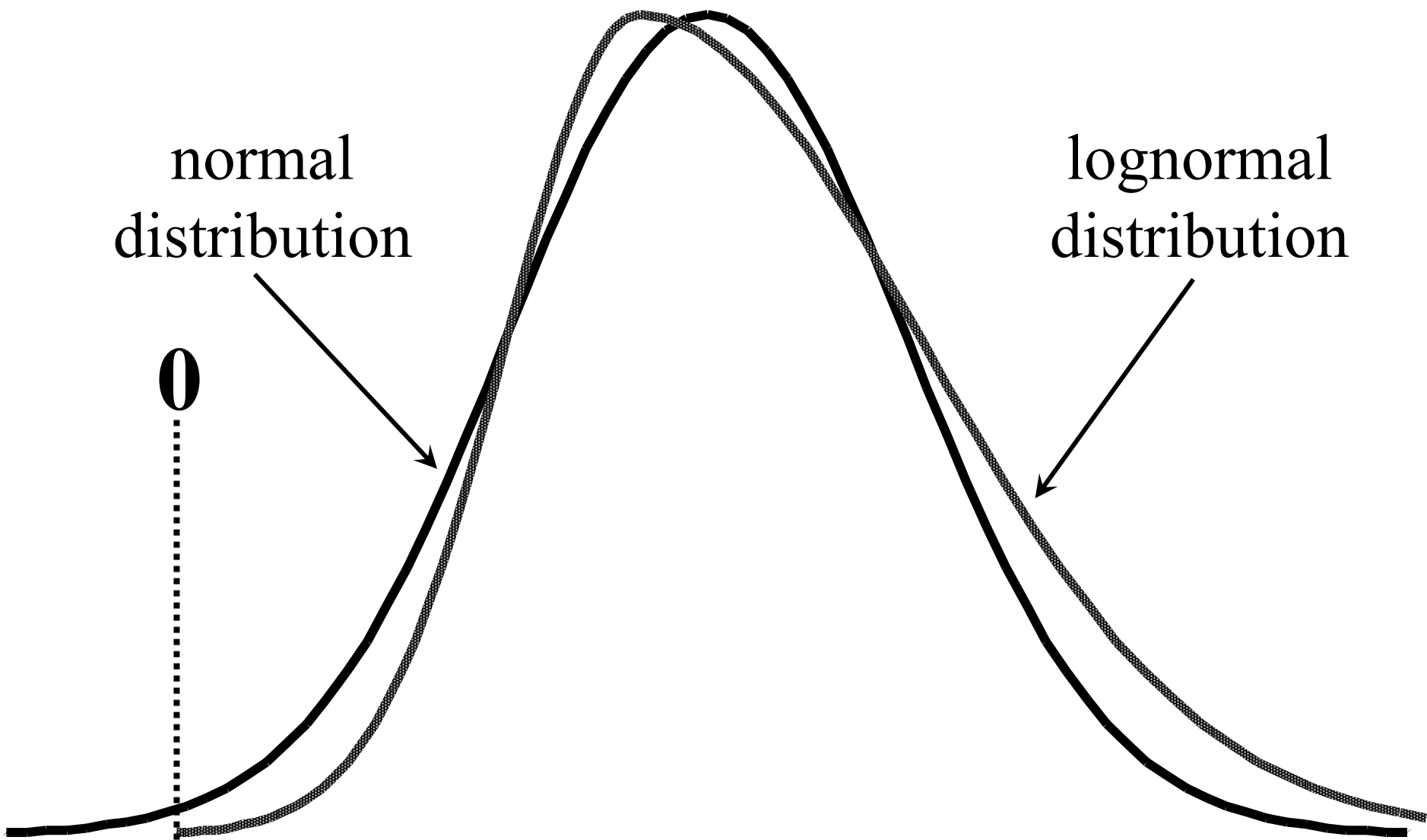
$-\infty$

$+\infty$

normal
distribution

lognormal
distribution

0



forward price = 100

	<u>normal</u> <u>distribution</u>	<u>lognormal</u> <u>distribution</u>	<u>price</u>
110 call	3.00	3.00	2.75
90 put	3.00	2.50	3.00

Are the options mispriced?

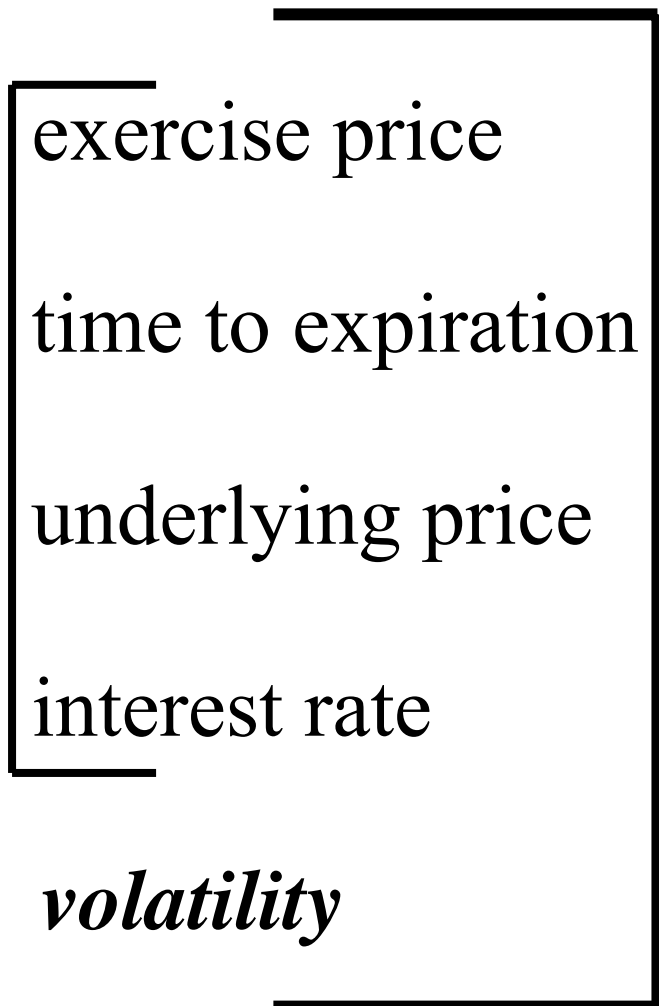
Why might they be priced the way they are?

realized volatility: The volatility of the underlying contract over some period of time (historical, future)

implied volatility:

derived from the prices of options in the marketplace

the marketplace's consensus forecast of future volatility



implied volatility

31%

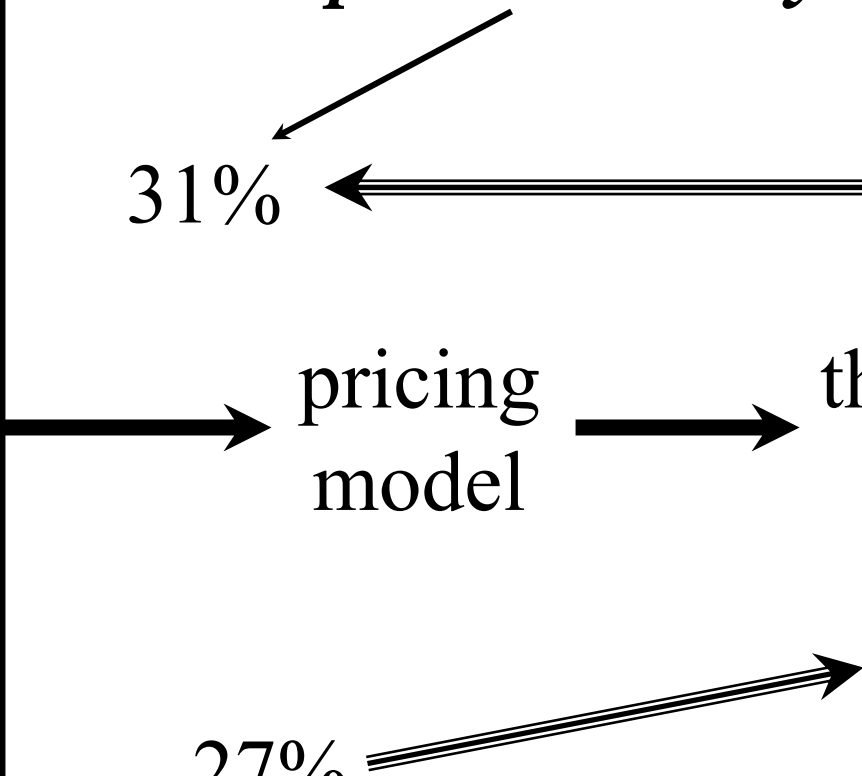
3.25

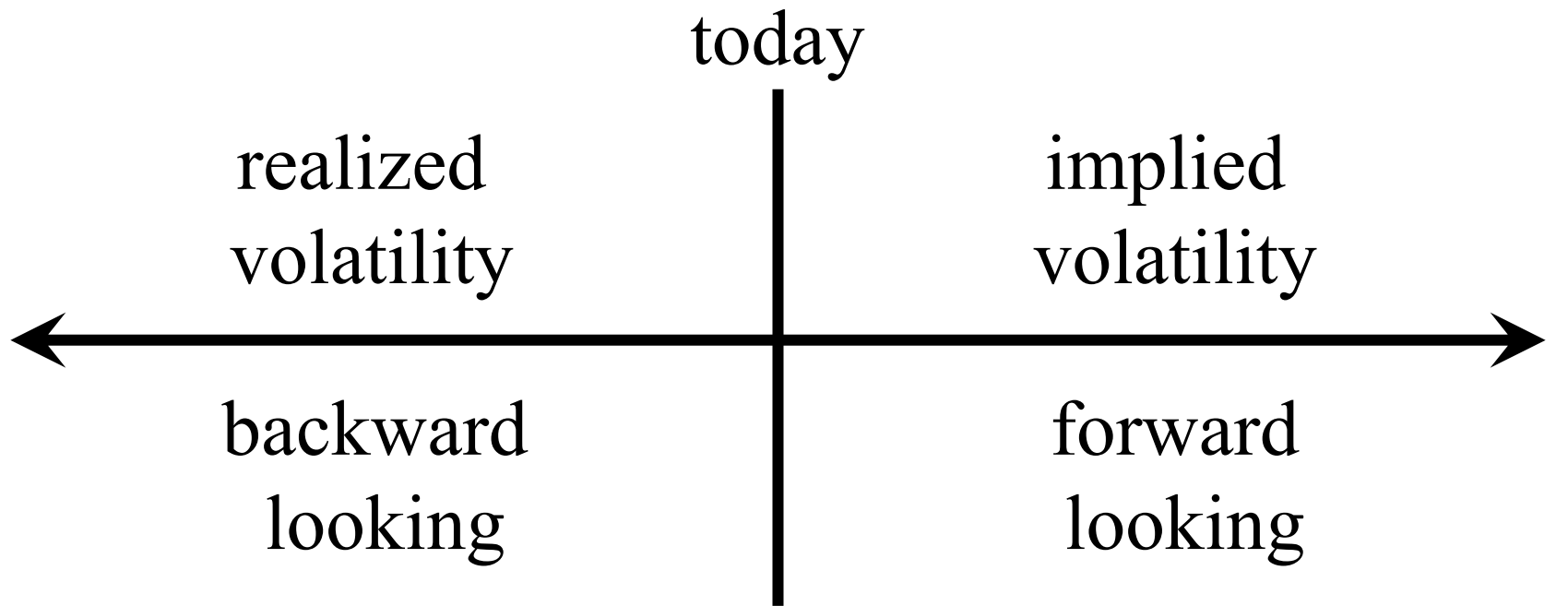
pricing
model

theoretical
value

27%

2.50





(what *has* occurred)

(what the marketplace
thinks *will* occur)

implied volatility = price

realized volatility = value

(historical, future)

Volatility characteristics

serial correlation

mean reversion

momentum

(G)ARCH – (generalized) auto-regressive
conditional heteroscedasticity

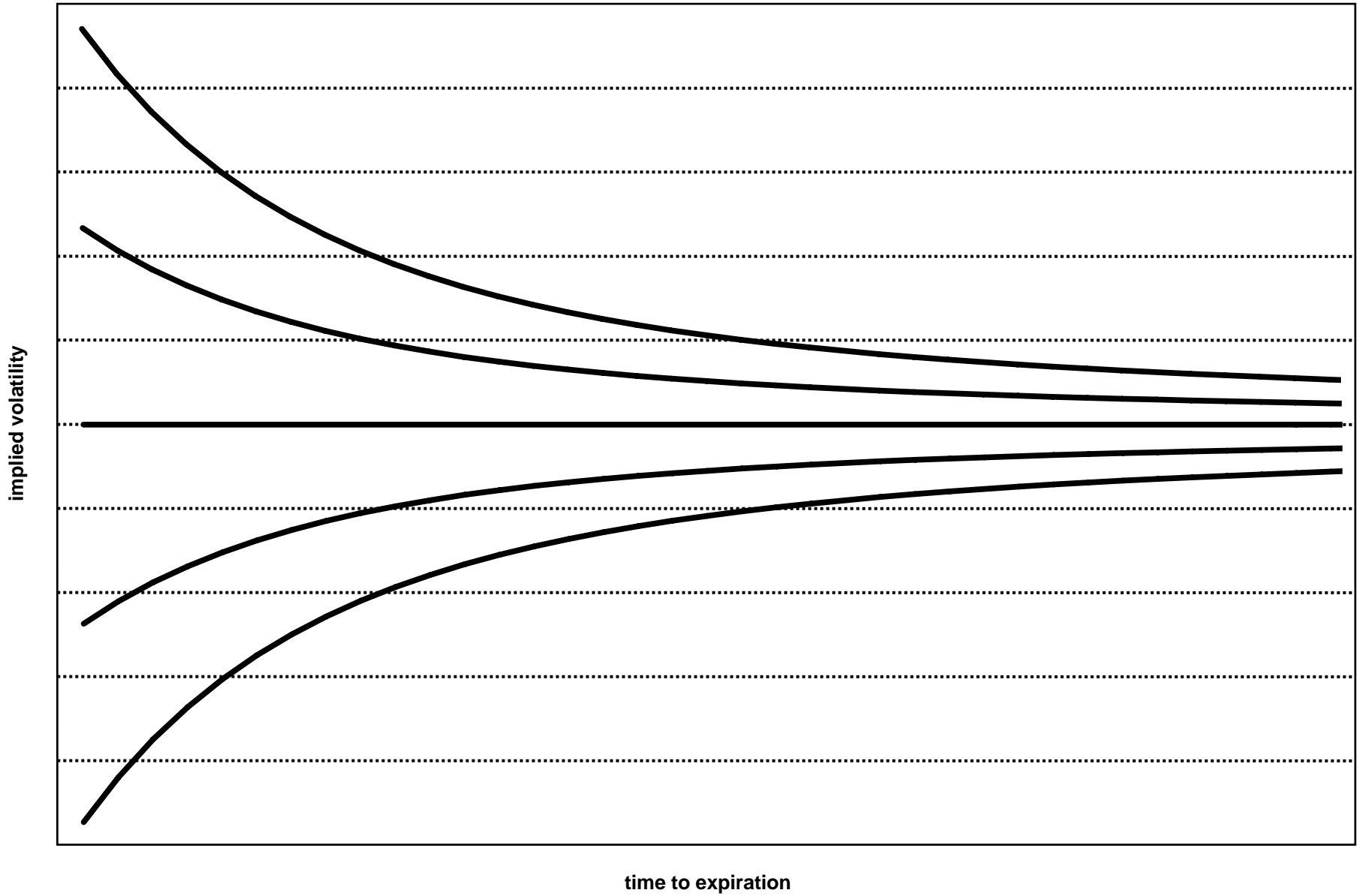
Changes in implied volatility

Average volatility = 35%

Jun implied	35%	45%	25%
Sep implied	35%	41%	29%
Dec implied	35%	37%	33%

Mean Reversion

Term Structure of Volatility



Volatility Exercise I (answers)

For each contract and volatility below, what would be an approximate daily and weekly standard deviation:

Stock at 43.50

	<u>25%</u>	<u>35%</u>	<u>45%</u>	<u>55%</u>
daily	.68	.95	1.22	1.49
weekly	1.51	2.11	2.71	3.32

Stock index 1242.00

	<u>30%</u>	<u>40%</u>	<u>50%</u>	<u>60%</u>
daily	23.29	31.05	38.81	46.58
weekly	51.75	69.00	86.25	103.50

Volatility Exercise II (answers)

For each contract, volatility, and time interval below, what would be an approximate one standard deviation price change:

Stock price = 77.15

volatility = 47%, time = 86 days **17.60**

volatility = 29%, time = 22 days **5.49**

Index price = 844.00

volatility = 39%, time = 5 weeks **102.07**

volatility = 15%, time = 11 weeks **58.23**