



Synthetics

Material prepared by
Sheldon Natenberg and Tim Weithers
Chicago Trading Co.
141 West Jackson Blvd.
Chicago, IL 60604
tel. 1 312 863 8000
sheldon.natenberg@chicagotrading.com
tim.weithers@chicagotrading.com

+1 December 100 call

-1 December 100 put

What will happen at expiration?

If stock > 100 : put is worthless / exercise call
buy stock at 100

If stock < 100 : call is worthless / assigned on put
buy stock at 100

+1 December 100 call

-1 December 100 put

Regardless of whether the underlying is above or below the exercise price at expiration:

buy the underlying at 100

Synthetic Long Underlying

long call + short put =

synthetic long underlying

short call + long put =

synthetic short underlying

	<u>delta</u>	<u>gamma</u>	<u>vega</u>
underlying contract	100	0	0

$$\text{abs}(\text{call delta}) + \text{abs}(\text{put delta}) = 100$$

$$\text{call gamma} = \text{put gamma}$$

$$\text{call vega} = \text{put vega}$$

long put + long underlying =

synthetic long call

short put + short underlying =

synthetic short call

long call + short underlying =

synthetic long put

short call + long underlying =

synthetic short put

Any strategy can be done using a synthetic equivalent.

Buy a December 100 straddle:

$$\left\{ \begin{array}{l} +1 \text{ December 100 call} \\ +1 \text{ December 100 put} \end{array} \right\}$$
$$\left\{ \begin{array}{l} +2 \text{ December 100 calls} \\ -1 \text{ underlying contract} \end{array} \right\}$$
$$\left\{ \begin{array}{l} +2 \text{ December 100 puts} \\ +1 \text{ underlying contract} \end{array} \right\}$$

Conversion

long underlying + synthetic short
underlying

long underlying + short call
long put

Reverse Conversion (Reversal)

short underlying + synthetic long
underlying

short underlying + long call
short put

Conversion (buy stock / sell call / buy put)

Credits: call price (C)
interest on call ($C*r*t$)
dividends (D)
exercise price (X)

Debits: stock price (S)
interest on the stock price ($S*r*t$)
put price (P)
interest on the put price ($P*r*t$)

$$C + C*r*t + D + X = S + S*r*t + P + P*r*t$$

Call Price - Put Price (*Combo*)

$$C + C * r * t + D + X = S + S * r * t + P + P * r * t$$

$$(C + C * r * t) - (P + P * r * t) = (S + S * r * t) - D - X$$

$$C * (1 + r * t) - P * (1 + r * t) = S * (1 + r * t) - D - X$$

$$(C - P) * (1 + r * t) = [S * (1 + r * t) - D] - X$$

$$(C - P) * (1 + r * t) = F - X$$

$$C - P = (F - X) / (1 + r * t)$$

Put-Call-Parity

Synthetic Relationship

Conversion Market

$$C - P = (F - X) / (1+r*t)$$

If $r = 0$; $D = 0$: $C - P = S - X$

sell December 100 call = 6.00

buy December 100 put = 2.25

buy underlying contract = ??? 103.75

$$6.00 - 2.25 = ??? - 100$$

stock price = 68.50

time to expiration = 4 months

interest rate = 6.00%

expected dividend = .22

65 combo?

$$C - P = (F - X) / (1+r*t)$$

$$F = 68.50*(1+.06*4/12) - .22 = 69.65$$

$$\begin{aligned} (F - X) / (1+r*t) &= (69.65 - 65) / (1+.06*4/12) \\ &= 4.65 / 1.02 \approx \mathbf{4.56} \end{aligned}$$

If we know any two of the call price, put price, or underlying price, we can always calculate the missing price:

$$C - P = (F - X) / (1+r*t)$$

$$C = (F - X) / (1+r*t) + P$$

$$P = C - (F - X) / (1+r*t)$$

$$F = (C - P)*(1+r*t) + X$$

$$S = (F + D)/(1+r*t)$$

If we know the prices of all the contracts, we can also calculate the *implied dividend* or the *implied interest rate*.

Synthetic Pricing Exercise For the questions below use the following relationships: $(C-P) = (F-X)/(1+r*t)$ and $F = S*(1+r*t) - D$

1. For each exercise price fill in the missing call or put price.

stock price = 64.20; time to expiration = 83 days

interest rate = 4.50%; expected dividend = 27¢

	<u>50</u>	<u>55</u>	<u>60</u>	<u>65</u>	<u>70</u>	<u>75</u>	<u>80</u>
call		10.98		5.04		1.92	
put	.58		3.08		8.53		16.37

2. What is the current stock price if... 25 call = 4.30 25 put = 2.85

time to expiration = 112 days; interest rate = 3.00%; expected dividend = 0

3. What is the implied index dividend if... 850 call = 26.80 850 put = 44.95

index price = 832.10; time to expiration = 44 days; interest rate = 2.00%

Synthetic Pricing Exercise (answers)

$$(C-P) = (F-X)/(1+r*t) \quad \text{and} \quad F = S*(1+r*t) - D$$

1. For each exercise price fill in the missing call or put price.

stock price = 64.20; time to expiration = 83 days

interest rate = 4.50%; expected dividend = 27¢

	<u>50</u>	<u>55</u>	<u>60</u>	<u>65</u>	<u>70</u>	<u>75</u>	<u>80</u>
call	15.02	10.98	7.62	5.04	3.17	1.92	1.27
put	.58	1.49	3.08	5.45	8.53	12.22	16.37

2. What is the current stock price if... 25 call = 4.30 25 put = 2.85

time to expiration = 112 days; interest rate = 3.00%; expected dividend = 0

26.22

3. What is the implied index dividend if... 850 call = 26.80 850 put = 44.95

index price = 832.10; time to expiration = 44 days; interest rate = 2.00%

2.30